Design of Lossless Junction, Given One Row in Its Scattering Matrix

It is desirable occasionally to find a lossless n-port junction which has a scattering matrix with a specified row. In some cases, it may be sufficient to know that such a junction exists. An example of how such a junction might be used is given later.

In order to illustrate how the desired junction can be designed and the corresponding scattering matrix found, a specified case is considered. The generalization to any other case is obvious. Since the numbering of the ports is arbitrary, it is assumed that the first row is given. For the specific case considered, the elements in the given row are

$$S_{1k} = S_{1k}\underline{/\theta_{1k}}, \qquad (k = 1, 2, 3, 4)$$
$$S_{15} = 0.$$

The equivalent circuit for the desired junction is shown in Fig. 1.

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shown in Fig. 1 does have the desired characteristics.

Next, consider the junction shown in Fig. 2. The junction is not necessarily lossless. Let the scattering matrix of this junction be denoted by (J). It is assumed that power is fed into port No. 1 and that the remaining ports are terminated in matched loads. The efficiency of the junction under these conditions is

$$\eta = (J_{12}^2 + J_{13}^2 + J_{14}^2)/(1 - J_{11}^2).$$

It is possible to design a lossless 4-port junction, which is called the *collector junction*, which combines the outputs of ports 2, 3, and 4 into a single output port. Let the scattering matrix of the collector junction be denoted by (C). The output port is designated the No. 1 (C) port. When the two junctions are connected, as shown in Fig. 3, the combination is a 2-port junction.

Let the elements in the first row of (C) have the values

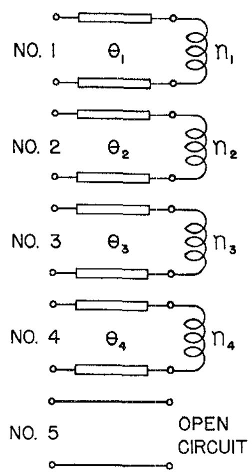

Fig. 1. An equivalent circuit for the desired junction.

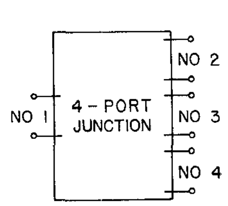

Fig. 2. An arbitrary junction.

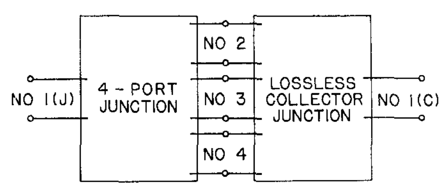

Fig. 3. Combination of arbitrary junction and collector junction.

The equivalent circuit consists of sections of transmission line and a multiwinding ideal transformer. The sections of transmission line produce the phase delays indicated, with

$$\theta_1 = -\theta_{11}/2,$$
$$\theta_k = -\theta_{1k} - \theta_1.$$

The turns ratios for the windings of the ideal transformer are

$$\frac{n_k}{n_1} = \frac{S_{1k}}{1 + S_{11}} \qquad (k = 2, 3, 4) \cdot$$

The writer will not bore the reader with the tedious details of verifying that the circuit

$$C_{11} = 0,$$
$$C_{1k} = J_{1k}^*/\sqrt{J_{12}^2 + J_{13}^2 + J_{14}^2}, \qquad (k = 2, 3, 4)$$

where the asterisk denotes the conjugate. The equivalent circuit for the collector junction is the same as the one shown in Fig. 1, omitting port No. 5. Now ports 2, 3, and 4 of the original junction are terminated in matched loads, and the efficiency of the 2-port junction has the value η given above. (It might appear from the last remark that all C_{kk} must be zero, but this is not true.) The voltage transmission coefficient for the 2-port network is

$$T = \sqrt{\eta}.$$

If port No. 1 (C) is terminated in a matched load, the voltage reflection coefficient at port No. 1 (J) is

$$\Gamma = J_{11}.$$

These remarks might be used to consider this question: *Is it theoretically possible to build a lossless antenna which collects all the incident power?* However, this question is too complex to be treated here.

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A Method of Calculating the Attenuation Constants of the Unwanted Modes in Mode Filters Using Resistive Sheets

Mode filters, which are one of the most important components in overmoded waveguides, give no attenuation to the main mode and large attenuations to some unwanted modes. This correspondence deals with mode filters using resistive sheets.

In these mode filters, the resistive sheets are mounted in a hollow waveguide so that they are always perpendicular to the electric field of the main mode, while the electric fields of the unwanted modes have tangential components along the resistive sheets.

In our discussion, the following three assumptions are postulated.

1) The thickness of the resistive sheet is much smaller than the skin depth, and its impedance is pure resistance. (In the millimeter wave region, a thin metal film deposited on a thin dielectric base is desired.)

2) The square resistance ($R_\square$) of the resistive sheet is large enough, and it does not disturb the field pattern of the waves in the hollow waveguide when it is inserted into the waveguide.

3) The attenuation of the unwanted modes is due to the heat loss in the resistive sheet according to the conduction current $J = \sigma E$ (σ = conductivity of the resistive material, E = tangential electric field along the resistive sheet).

Under the above assumptions, if the resistive sheets are inserted into a straight waveguide and have the same cross section along the guide axis, the attenuation constant of any unwanted mode may be given approximately by

$$\alpha = 8.686 \frac{1}{2P} \iint \frac{1}{2\sigma} |J|^2 dS \quad \text{dB/m}$$

P = transmission power of any unwanted mode

where the surface integral is taken over the transverse cross section of the resistive sheets of thickness t, and its value is equal to the power losses in the resistive sheets per unit length along the axial direction.

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Using the relations of $J=\sigma E$ and $R_\square = 1/\sigma t$, the above equation becomes

$$\alpha = 8.686 \frac{1}{4R_\square P} \int |E|^2 ds \quad \text{dB/m} \qquad (1)$$

E = tangential electric field along the resistive sheets of any unwanted mode.

In this case, the line integral is taken over the curve of the transverse cross section of the resistive sheets. Since $R_\square$ is large enough, it is permissible to use for P and E the values of a hollow waveguide. Hence, they are given directly by the Handbook, and $|E|^2 = |E_t|^2$ (E_t = transverse component of the electric field along the resistive sheets) for the TE mode, while $|E|^2 = |E_t|^2 + |E_z|^2$ (E_z = axial component of the electric field along the resistive sheets) for the TM mode.

At first, the attenuation constant of the dominant TE_{10} mode was calculated in a standard rectangular waveguide with a resistive sheet parallel to the E plane at the center of the H plane in order to evaluate the degree of approximation. Figure 1 shows the relation between the value of $R_\square$ and the theoretical values of the attenuation constant at 50 Gc/s for a WRJ-500 (WR 19) waveguide ($a = 4.78$, $b = 2.39$ mm). The solid curve shows the exact value found by the transverse resonance method,[1] while the dotted curve shows the approximate value calculated by (1). These curves are almost coincident over 300 Ω.

Second, let us study a circular waveguide mode filter having radial resistive sheets through which the circular electric waves (TE_{0n}) can pass with no loss.

In circular waveguides, there are two degenerated modes which have circumferential field variations of cos ($m\phi$) and sin ($m\phi$); therefore, it is an important problem to determine the number and the distribution of the resistive sheets. An ideal mode filter must have at least $4m$ resistive sheets on the radial lines equally spaced every $90/m$ degrees, in order to give an equal attenuation to the TE_{mn} or TM_{mn} ($m \neq 0$) mode having a different incident angle of polarization at the input of the mode filter. Figure 2 shows the ideal mode filters for several unwanted modes.

In this case, the attenuation constant of the TE_{mn} mode per one sheet on the radial line is given by the following equation using (1):

$$\alpha = 8.686 \frac{m^2 \chi_{mn}'}{\pi(\chi_{mn}'^2 - m^2)\{J_m(\chi_{mn}')\}^2} \cdot \frac{\lambda_{gmn}}{\lambda R R_\square} \int_0^{\chi_{mn}'} \left\{\frac{J_m(\xi)}{\xi}\right\}^2 d\xi \quad \text{dB/m} \qquad (2)$$

λ_{gmn} = Guide wavelength of the TE_{mn} mode
λ = Free space wavelength
R = Radius of the circular waveguide
ζ = Free space characteristic impedance (377Ω)
χ_{mn}' = The nth root of $J_m' = 0$ except zero.

If the sheets of the number of p are mounted on the radial lines equally spaced $360/p$ degrees, (2) must be made $p/2$ times for $p > 2m$. For the particular case when $p = 2m$, the attenuation of the TE_{mn} is varied by the incident angle of polarization at the input of the mode filter; therefore, (2) must be made $2m$ times for the maximum attenuation and zero attenuation for the minimum attenuation.

[1] H. Buseck and G. Klages, "Das homogene Rechteckrohr mit Dampfungsfolie," *A.E.Ü.*, vol. 12, pp. 163–168, April 1958.

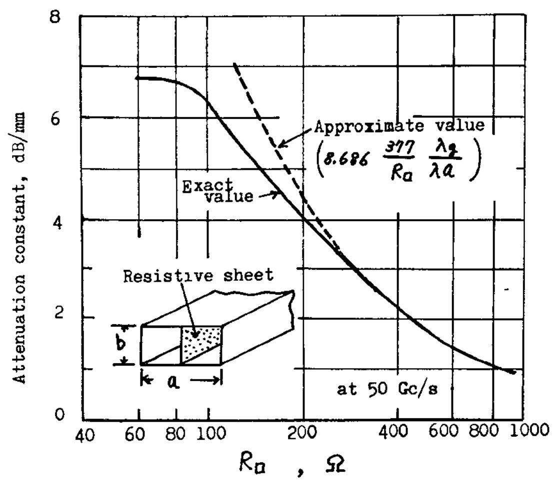

Fig. 1. Theoretical attenuation constants of the dominant mode in the standard rectangular waveguide (WRJ-500) having resistive sheet.

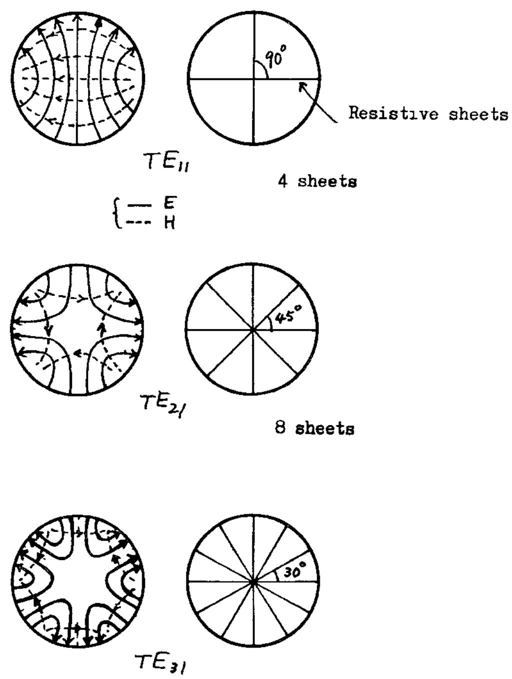

Fig. 2. Desired distribution of the resistive sheets for three unwanted modes.

Figure 3 shows an experimental model ($2R = 12.3$ mm) for attenuating the TE_{11} mode with a polarization parallel to the resistive sheet. The resistive sheet ($p = 2$), which consists of chrome deposited on a Mylar base (0.08 mm thick), is supported by polystyrene foam (70 mm long). The experimental values of the attenuation vs. the length l (Fig. 3) are plotted in Fig. 4 for $R_\square = 400$, 600, 1000Ω (dc measured). The dotted straight lines show the theoretical attenuation constants calculated by (2).

The slope of the experimental values is almost coincident with the theoretical attenuation constant for each case. The attenuations at $l = 0$ correspond to those of the taper sections of the resistive sheets.

The experimental attenuations of the TE_{11} mode having a polarization perpendicular to the resistive sheet and the TE_{01} mode are nearly zero. [The theoretical heat losses at the guide walls of the TE_{11} and the TE_{01} modes are 0.011 and 0.0093 dB per 100 mm, respectively, and the dielectric loss of the polystyrene foam ($\epsilon_r = 1.03$, $\tan\delta = 10^{-4}$) is 0.034 dB per 70 mm for the TE_{11} mode.]

The same experiments as the above were made for the TE_{21} mode in another model, and good results were obtained.

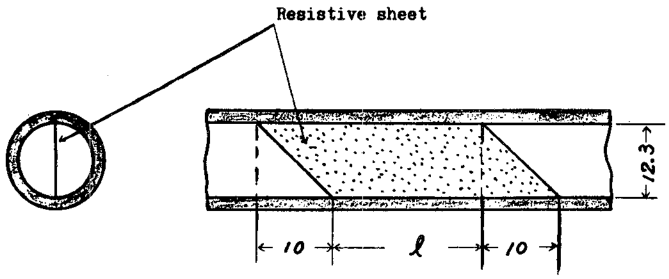

Fig. 3. Mode filter for attenuating the TE_{11} mode having a polarization parallel to the resistive sheet. Dimensions are in millimeters.

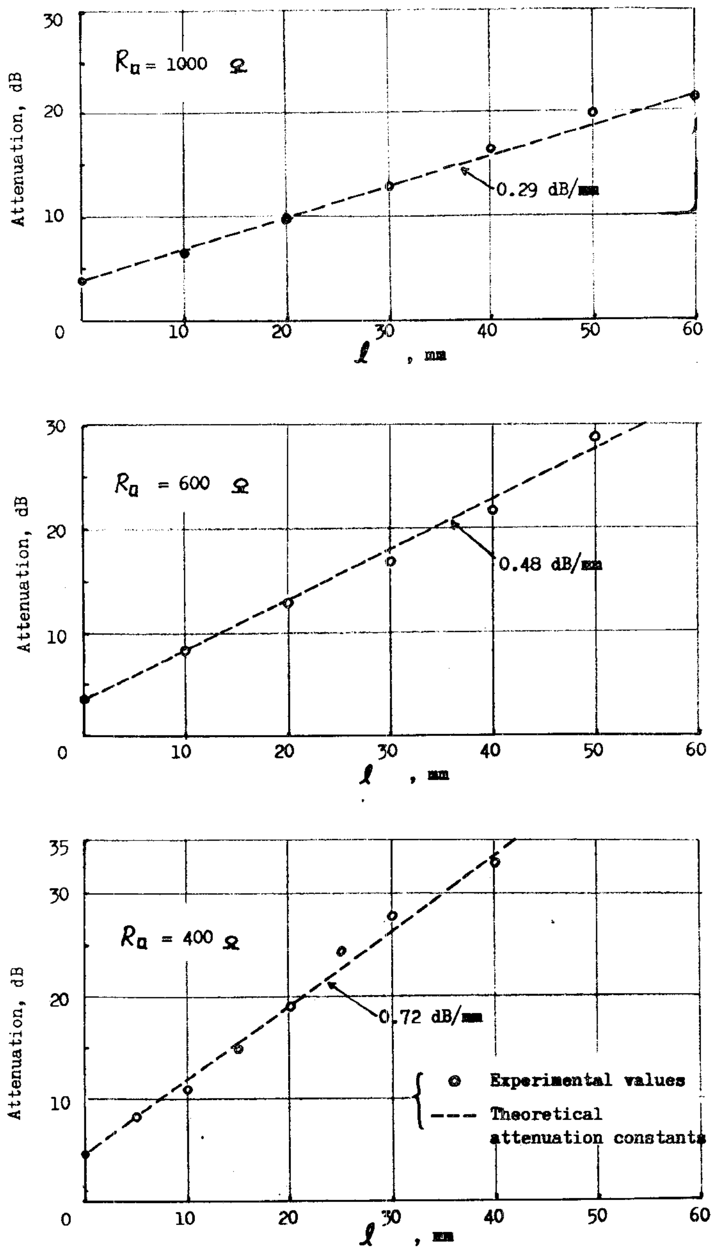

Fig. 4. Attenuations of the TE_{11} mode in the mode filter of Fig. 2. The frequency is 50 Gc/s.

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Calibration of Coaxial Bolometer Mounts

The Radio Standards Laboratory announces that an additional service is now available for the measurement of the calibration factor[1] of nominal 50-ohm coaxial bolometer units. The new service provides for calibration at 3 GHz, in addition to the frequencies of 100 MHz and 1 GHz that have been available for a number of months. Measurements are made of the 1- and 10-milliwatt power levels only, with no provision at present for the calibration of bolometer-coupler units.

The limit of uncertainty in determining the calibration factor at 3 GHz is within 1.5 percent for well-designed bolometer units. Limits of uncertainty may be greater for bolometer units having a VSWR higher than 1.1. The service includes the calibration of both barretter and thermistor types of bolometer units having operating resistances of 50, 100, and 200 ohms.

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[1] The calibration factor of a bolometer unit is defined as the ratio of the substituted dc power in the unit to the RF power incident upon the bolometer unit.

Rectangular Waveguide Short Circuit with Cylindrical Slugs

The design of precision waveguide choke shorts imposes many problems, especially as concerns millimeter wave devices. A guide consisting of a rectangular waveguide with a cylindrical rod placed along its axis may help to solve the problem. Such arrangements are used in coaxial-to-strip line adapters [1], [2]. Adequate electric characteristics can be achieved in very simple mechanical designs of shorts comprising cylindrical sections of low and high impedance [3]. To design such a short it is necessary to know, however, the wavelength in the rectangular waveguide comprising a cylindrical rod—a problem which still appears to be unsolved.

The cutoff wavelength of the TE_{11} mode excited in a waveguide with a rod placed inside may be calculated from the empirical equation

$$\frac{\lambda_c}{2a} = 1 + \frac{1}{2}\left[\left(\frac{D}{b}\right)^2 + 1\right] \cdot \left[\frac{a}{2b}\left(\frac{D}{b}\right)^2 + 0.1\frac{D}{b}\right]$$

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